The exceptional sets on the run-length function of beta-expansions

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Abstract. Let $\beta > 1$ and the run-length function $r_n(x,\beta)$ be the maximal length of consecutive zeros amongst the first n digits in the β-expansion of $x \in [0,1]$. The exceptional set

$$E_{\max}^{\varphi} = \left\{ x \in [0,1] : \liminf_{n\to\infty} \frac{r_n(x,\beta)}{\varphi(n)} = 0, \limsup_{n\to\infty} \frac{r_n(x,\beta)}{\varphi(n)} = +\infty \right\}$$

is investigated, where $\varphi : \mathbb{N} \to \mathbb{R}^+$ is a monotonically increasing function with $\lim\limits_{n\to\infty} \varphi(n) = +\infty$. We prove that the set $E_{\max}^{\varphi}$ is either empty or of full Hausdorff dimension and residual in $[0,1]$ according to the increasing rate of φ .